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Geometric error measurement and compensation of machines—An update

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ABSTRACT

For measuring machines and machine tools, geometrical accuracy is a key performance criterion. While numerical compensation is well established for CMMs, it is increasingly used on machine tools in addition to mechanical accuracy. This paper is an update on the CIRP keynote paper by Sartori and Zhang from 1995 [Sartori S, Zhang GX (1995) Geometric error measurement and compensation of machines, *Annals of the CIRP* 44(2):599–609]. Since then, numerical error compensation has gained immense importance for precision machining. This paper reviews the fundamentals of numerical error compensation and the available methods for measuring the geometrical errors of a machine. It discusses the uncertainties involved in different mapping methods and their application characteristics. Furthermore, the challenges for the use of numerical compensation for manufacturing machines are specified. Based on technology and market development, this work aims at giving a perspective for the role of numerical compensation in the future.

1. Introduction

Machine tools and measuring machines with 3–5 axes can be found in vast numbers and all areas of modern production: from automotive to aerospace, from the production of consumer goods to medical goods. The trend towards individualized goods and smaller lot sizes increases the importance of machine flexibility in production. Instead of relying on single purpose machines, manufacturing is increasingly organized in production cells, that can be adapted to multiple and changing products. The world market for machine tools in 2007 is estimated to be 71 billion US dollars, which represents a growth of 18% compared to 2006 [51]. Fig. 1 shows the top 10 countries of machine tool producers and their machine tool consumption.

One of the main performance criteria for a modern production cell is its ability to manufacture accurate parts. This can only be achieved by a controlled and deterministic manufacturing process. While the repeatability of the machine is a necessary requirement for a well-controlled process, the geometric accuracy of the part can be achieved either by a feedback loop through part metrology [102] or by accurately calibrated machine tools [116]. Due to shorter product life cycles and small series production, the absolute accuracy of machine tools is of increasing importance. Short-production ramp up times do not allow an iterative optimisation of the product quality. McKeown introduced the term *volumetric accuracy* to define the ability of a machine to produce accurate 3D shapes [107]. *Volumetric accuracy* minimizes the ramp up cost for new or changing processes. Volumetric accuracy of machine tools and co-ordinate measuring machines (CMMs) has to be assured by precise and traceable metrology. The information gained may be used to characterize the machine or to increase the accuracy by numerical compensation.

To perform an error mapping and a subsequent compensation of geometric errors requires an understanding of the sources and the effects of geometry errors in machines and calibration procedures. Significant work towards volumetric calibration was done in the production research community in the last century, with many CIRP contributions shaping this field of research. A CIRP keynote paper by Sartori and Zhang [116] summarizes the general concepts and the available technology at that time. This paper will give an update on the 1995 paper. While the paper by Sartori and Zhang reviewed the foundations of numerical compensation, this paper will focus on the practical application. Furthermore, it takes into account the appearance of new calibration methods, new concepts in international standards and the growing capabilities of machine tool controllers to compensate mathematically geometry errors.

2. Sources of geometry errors

The accuracy of machine tools and CMMs is affected by many error sources. These error sources may cause a change in the

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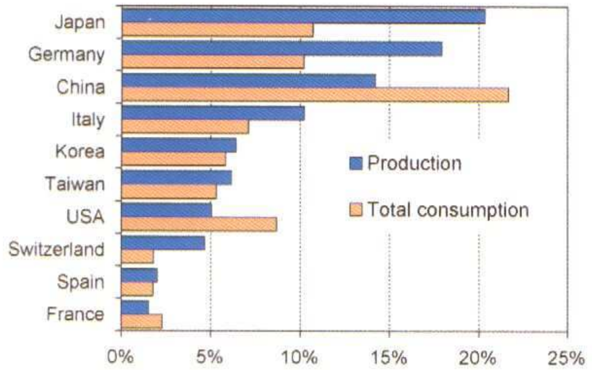


Fig. 1. The top 10 machine tool producing countries (2007) [51]: national production and consumption of machine tools (in US$, percentage of world total).

geometry of the machine's components present in the machine's *structural loop*. According to ANSI and ASME standards [4] a structural loop is defined as an assembly of mechanical components which maintain a relative position between specified objects. In a machine tool, the structural loop includes the spindle shaft, the bearings, the housing, the guideways and frame, the drives and the tool and work-holding fixtures. Due to a change in geometry of these structural loop components, the actual end-effector position and orientation relative to the workpiece differs from its nominal position and orientation, resulting in a relative orientation and positioning error. The magnitude of this positioning and orientation error depends on the sensitivity of the machine's structural loop on various error sources.

The following reported major error sources affect the accuracy of the relative end-effector position and orientation [58,89,127,130,154]:

- Kinematic errors;
- Thermo-mechanical errors;
- Loads;
- Dynamic forces;
- Motion control and control software.

In precision instruments and machines many parts interact to achieve a final accuracy. Each part contributes to the total accuracy due to deviations caused by the above-mentioned effects. Although in practice the interaction between these effects plays an important role in the overall system behaviour, here we will focus on these effects separately [121].

2.1. Kinematic errors

Kinematic errors are errors due to imperfect geometry and dimensions of machine components as well as their configuration in the machine's structural loop, axis misalignment and errors of the machine's measuring systems [1,41,28,89,45,134,158,126,127,116,130,8,128,2,121,95,44,62,96,48,123,153,12,17,46,118,90,19,20,166,170,171]. If the position of one axis influences the location and component errors of another, then the single errors of this axis are functions of the axis under test and of the influencing axis. Furthermore location errors might also become functions of axis positions. In principle, the systematic of kinematic errors stays the same, but the error functions become more complicated. The kinematic structure of machines and the corresponding errors including uncertainty evaluation, respectively will be dealt with extensively in Sections 3–5.

2.2. Thermo-mechanical errors

Due to the presence of, sometimes changing, internal and external heat/cold sources in machine tools and CMMs and the very often significant expansion coefficients and expansion coefficient differences of machine part materials, the resulting thermal distortion of the machine's structural loop often dominate the accuracy of an executed task [130,13,49,42,54,27,29,26,161,171]. Expansion coefficient differences may lead to thermal stresses if rules of exact constraint design have not been met carefully [121]. Changed thermal conditions may cause location and component errors of machines. This introduces another complexity for error functions and will be described in Sections 4 and 5, but again does not change the systematic of the geometric error description.

2.3. Loads

If a machine exhibits non-rigid body behaviour, location errors and component errors change due to internal or external forces [121]. In some cases, the weight and position of for instance the workpiece or moving carriages of the machine, or the static machining—or measuring forces can have a significant influence on the machine's accuracy due to the finite stiffness of the structural loop. Schellekens et al. [121] and Spaan [130] have shown that these errors can be significant when compared to the kinematic errors of a machine tool or CMM. For instance, if straight guideways bend due to the weight of the moving slide, the slide will show a vertical straightness and a pitch error motion. This is called "quasi-rigid behaviour". Such effects will be caught by measuring the error motions and do not change the systematic of the error description. This will be described in Sections 4 and 5.

2.4. Dynamic forces

The trajectory to be realized by a machine tool or a CMM is also affected by the dynamic behaviour of the machine's structural loop. In this case (rapidly) varying forces such as machining forces, measuring forces or forces caused by accelerations or decelerations should be considered instead of quasi-static ones. Vibrations may result in a deformation of the structural loop of the machine under consideration. The deformations due to vibrations in the structural loop are often hard to compensate. This is due to the very often unknown amplitude and in particular the phase angle of the vibration frequencies. This contributes to the uncertainty of the tool/probe position relative to the workpiece. Relevant information concerning deviations due to dynamic forces can be found in [154,115,60,157].

Motion control and control software effects on the geometrical error can be significant. In the analysis, they are often distinguishable from the errors caused by other error sources by applying different feed speeds and/or accelerations for the same motion path.

However, precision machining or measuring is often carried out at small feed speeds, with small acceleration and decelerations as well as small cutting/measurement forces. Error correction and compensation, without taking these dynamic forces into account, can, nevertheless, be successful in these cases. In the following, this keynote paper will focus on statical geometrical errors of machine tools and CMMs. This is done by assuming that an error model can include major thermo-mechanical and non-rigid body effects, but cannot include all errors caused by dynamic forces in moving machines.

3. Kinematic structures of machines

Kinematics, derived from the Greek word *kinema—movement*, teaches the knowledge and mathematical descriptions of the movement of bodies and particles in space [147]. This movement is described, e.g. by the position in 3D space as well as acceleration and speed of a rigid body system without regards to the forces which act upon it [53,111]. The position of a point in space is generally described in a three-dimensional co-ordinate system. Cartesian, cylindrical or spherical co-ordinate systems are distinguished (Fig. 2) and transformations can be applied to switch from one system to any other [111,114]. Linear and rotary

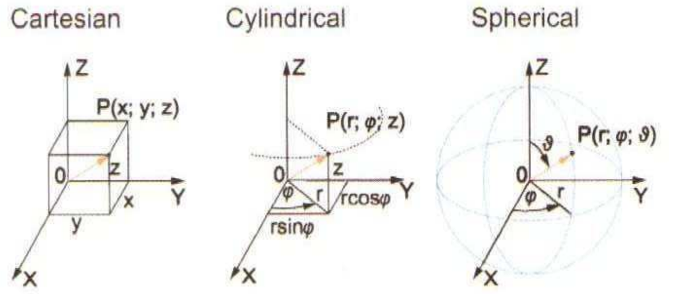


Fig. 2. Co-ordinate systems (Cartesian, cylindrical and spherical).

joints have to be arranged in order to achieve the required movement of the components belonging to machines, robots or instruments. A machine structure includes the supporting frame of a machine.

Functional elements of a machine, e.g. the drives and guides, are added to this frame. The kinematic structure is defined by the layout of machine components and their axes. With some abstraction, kinematic structures can be illustrated as shown in Fig. 3. The kinematic chain shows all axes, the workpiece, the tool and the bed of the machine. It symbolizes the flow of the movement in a kinematic structure. Along the kinematic chain, position and orientation of the tool centre point (TCP) can be calculated at one end of the chain. Serial and parallel structures can be distinguished. The characteristic of serial structures is that all axes can be moved independently (Fig. 3).

The kinematic chain of a serial structure is only closed during the manufacturing or measuring process by touching the part's surface [18,120]. Most machine tools and measuring machines have a serial structure. A notation based on Schwerd [164] can be used to describe the serial kinematic structure from the tool to the workpiece (Fig. 4) [21,58]. Parallel structures have a kinematic chain where a single machine component is driven by two or more independently controlled drives [151]. The most popular example is the so-called Steward/Gough-platform [52,132]. The advantages of parallel setups can be higher agility and stiffness. But on the other hand, these systems are more complex regarding movement control and may have strongly varying static and dynamic properties in the working volume [136,108,57,159].

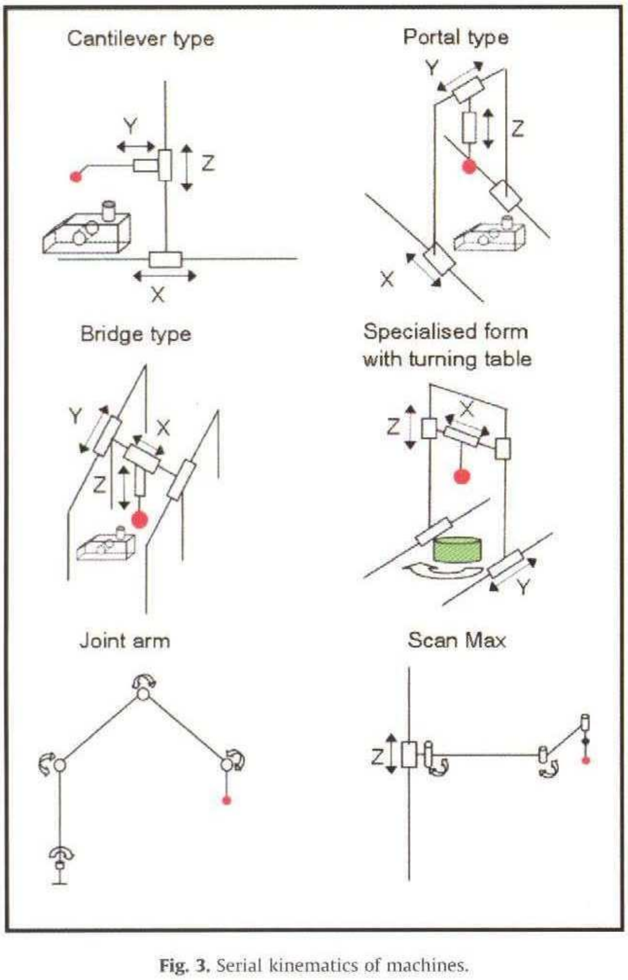


Fig. 3. Serial kinematics of machines.

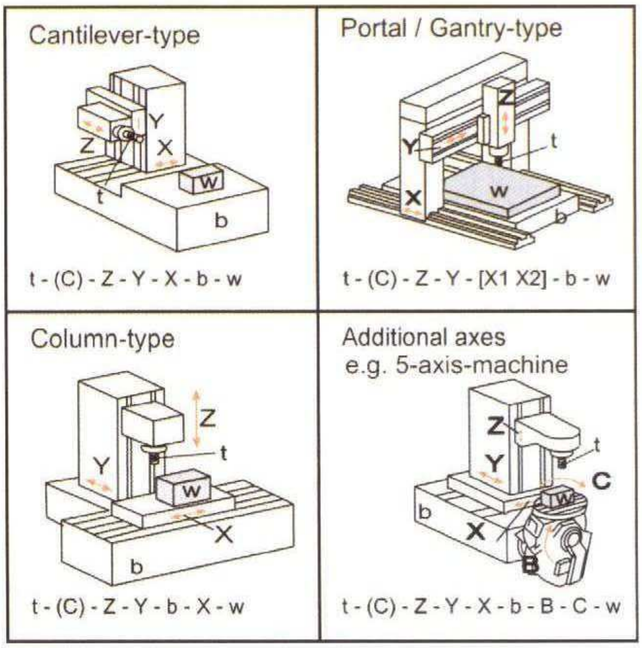


Fig. 4. Serial kinematics abstracted to kinematic chain (t: tool; b: bed; w: workpiece) [164].

4. Description of geometric errors

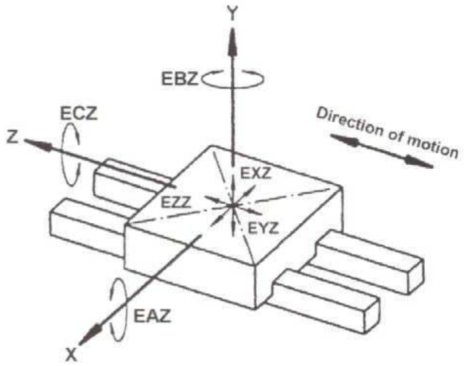
Relevant errors of a machine tool are relative error motions between the tool and the workpiece [117]. For a CMM, the relative motion between workpiece and probe has to be observed. The description of component errors starts with the assumption of the machine's rigid body behaviour. Each movement of a machine axis can be described by six degrees of freedom: three translations and three rotations, whereas generally only one degree of freedom is the nominal movement and the desired motion of the linear or rotational axis.

The notation of an axis movement is standardized in ISO 841 [87]: X, Y, and Z denote the linear movements, A, B, and C the rotations around X, Y, and Z, respectively. If there are more axes, then they are numbered (e.g. $X1$, $X2$, $X3$) or called U, V, W (linear axes) or D, E (rotational axes). Although other notations exist, e.g. VDI 2617-3 [146], the ISO notation will be used throughout this paper.

4.1. Component errors

For a nominally linear movement, the six component errors are the positioning error, two straightness error motions, roll error motion and two tilt error motions, which are called pitch and yaw error motion for horizontal axes. According to [81], straightness errors do not have a linear term; therefore orientation errors (e.g. squareness and parallelism) are considered separately, see Section 4.2. Fig. 5 describes these six component errors for a horizontal Z movement. Under the assumption of rigid body behaviour, these errors are functions of the nominal movement only and do not depend on the location of the other axes.

For a nominal rotational movement, the six component errors are two radial error motions, one axial error motion, the angular position error, and two tilt error motions. Fig. 6 shows these component errors for a C movement.

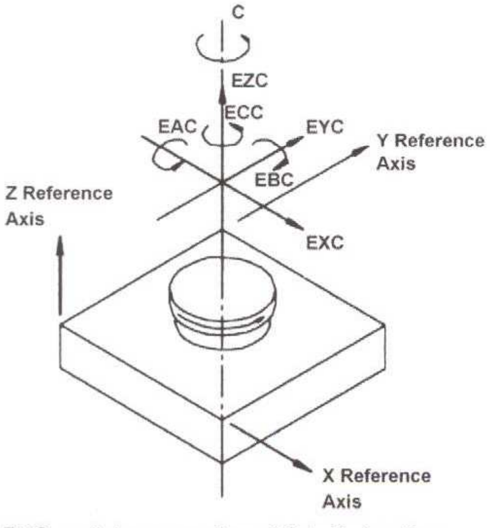


EXZ: horizontal straightness error motion of Z
(straightness error motion of Z in X direction)
EYZ: vertical straightness error motion of Z
(straightness error motion of Z in Y direction)
EZZ: positioning error
EAZ: pitch error motion of Z (tilt error motion around X)
EBZ: yaw error motion of Z (tilt error motion around Y)
ECZ: roll error motion

Fig. 5. Component errors of horizontal *Z* axis according to ISO 230-1 [81]. EXZ: horizontal straightness error motion of *Z* (straightness error motion of *Z* in *X* direction); EYZ: vertical straightness error motion of *Z* (straightness error motion of *Z* in *Y* direction); EZZ: positioning error; EAZ: pitch error motion of *Z* (tilt error motion around *X*); EBZ: yaw error motion of *Z* (tilt error motion around *Y*); ECZ: roll error motion.

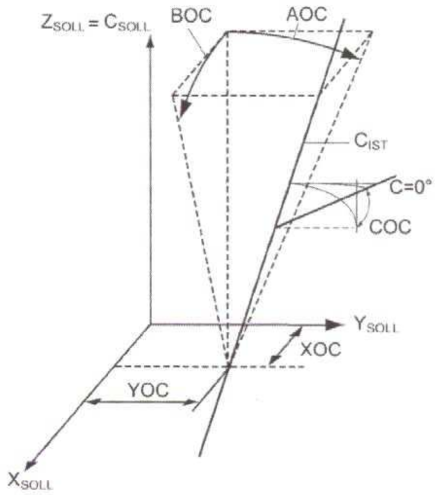
4.2. Setting up a machine co-ordinate system

The co-ordinate system of a machine should not be defined by machine components, but by the movements of the axes. A practical way of defining the co-ordinate system of a serial kinematic machine is as follows: one movement defines the primary direction of the co-ordinate system. A second movement will define the secondary direction of the co-ordinate system, i.e. the rotation around the primary direction. The positions of axes define the origin of the co-ordinate system [100]. The location (position and orientation) of other machine axes and components like machine table, alignment slots, centre bores on rotary tables, are given relative to the machine co-ordinate system. The following section describes the definition of location errors.



EXC: radial error motion of C in X direction
EYC: radial error motion of C in Y direction
EZC: axial error motion of C
EAC: tilt error motion around X of C
EBC: tilt error motion around Y of C
ECC: angular positioning error

Fig. 6. Component errors of *C* axis according to ISO 230-7 [85]. EXC: radial error motion of *C* in *X* direction; EYC: radial error motion of *C* in *Y* direction; EZC: axial error motion of *C*; EAC: tilt error motion around *X* of *C*; EBC: tilt error motion around *Y* of *C*; ECC: angular positioning error.



X0C: X position of C, Y0C: Y position of C
A0C: squareness of C to Y, B0C: squareness of C to X
C0C: zero angular position of C

Fig. 7. Location errors of *C* axis average line. XOC: *X* position of *C*; YOC: *Y* position of *C*; AOC: squareness of *C* to *Y*; BOC: squareness of *C* to *X*; COC: zero angular position of *C*.

4.3. Location errors

The location error of an axis (linear or rotational) is defined as an error from the nominal position and orientation of this axis in the machine co-ordinate system. Since the axis motion itself will show motion errors over the travel, generally the average line is defined as the nominal axis for definition of the location errors. This location of a rotational axis in regard to the nominal position is expressed by five location errors: two position errors, two orientation errors, and – in analogy to the zero position of a linear axis – the zero angular position. Fig. 7 shows these five location errors for a *C* movement.

A linear motion axis is defined by a vector with a zero position on the vector. Therefore there are just three location errors for a linear movement: two orientation errors and the zero position. Fig. 8 shows the location errors for a *Z* movement.

5. Mapping of geometric errors

Analyzing the geometrical errors of a machine, the relevant parameters and the most suitable measuring method depend on the machine geometry and the purpose of this evaluation. It can be distinguished between "direct" and "indirect" methods for detecting geometric errors in machines. In this paper, "direct" measuring means the analysis of single errors. In contrast to this, "indirect" measuring implies methods which focus on superposed errors. Both groups are considered in the following.

5.1. Direct measurement

"Direct" measurements allow the measurement of mechanical errors for a single machine axis without the involvement of other axes. Direct measurements can be classified in three separate subgroups based on their metrological reference: the material-based methods use artefacts, such as straightedges, line scales or step gauges. The laser-based methods use the laser light's linear propagation and its wavelength as a reference. The gravity-based methods measure in reference to the gravity field of the earth [10,56]. The general characteristics of these three principles will be briefly discussed in the following.

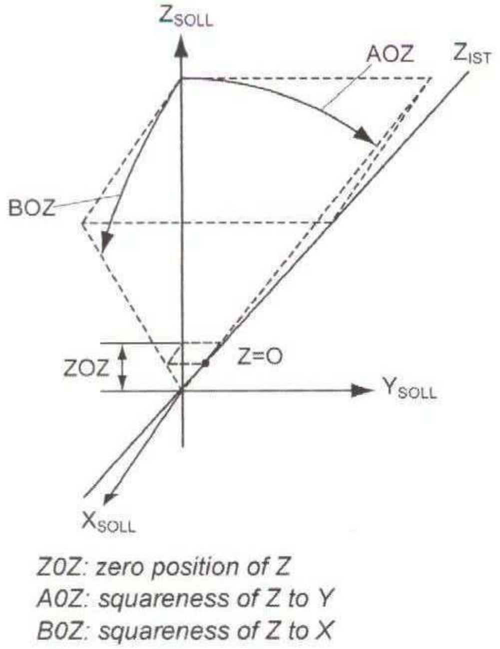


Fig. 8. Location errors of a Z movement. ZOZ: zero position of Z; AOZ: squareness of Z to Y; BOZ: squareness of Z to X.

Material standards for error mapping are primarily subject to their dimensions and material, because these items influence the uncertainty of the measuring results as long gauges can bend forced by gravity and metal materials age and change dimensions with time [91,152]. Special attention is paid to the calibration uncertainty of the material standard.

Multidimensional artefacts, such as ball plates, have gained widespread use in recent years, as they helped to overcome the drawback of elementary material standards which typically represent only one special use in terms of dimension or measuring task [152,158,103,105,81,116].

A number of laser-based measurement methods, such as the laser interferometer, are applied to machine calibration tasks as the laser beam is particularly suitable for length measurement. Due to its long-coherence length, the use of interferometric techniques for high-precision measurements is possible even for long axes. These methods principally measure the machine-positioning properties. Some measurement systems combine multiple sensors for simultaneous measurement of positioning, straightness and angular errors [81,99].

Certain error influences must be considered when applying laser interferometer-based-techniques for machine calibration. For interferometric methods, errors in the laser wavelength are transferred directly into errors in the length measurement [114]. The laser wavelength may change or be different from nominal, because of errors in the frequency stabilization [33,101].

The environmental factors, however, have a non-negligible impact: air temperature, pressure, density and humidity – as stimulated by temperature, locally leaking gases like CO_2 or evaporating solvents – influence the wavelength compensation to a significant extent. A deflection of the laser beam can be caused by temperature changes and gradients in the machine because the light is refracted by turbulences and inhomogeneity of the air. This results in possible errors of straightness and position measurements. Due to varying densities, turbulences also cause uncertainties in the optical path length of the laser beam, which must be considered for high-precision applications [23,25,38]. Even when the major heat load may occur from the drives, the heat output of the laser system can influence the measurements as typical helium–neon lasers emit over 5 W of heat. In small, high-precision machines, this can lead to local temperature gradients in the machine resulting in deviations and thus errors in the machine calibration [16,113].

The gravity-based methods use the direction of the gravity vector as a metrological reference. Typical examples for such measuring devices are inclinometers and spirit levels (mechanical or electronic). They allow the measurement of angular motion errors around horizontal axes. Not measurable are angular motions around the vertical axis. In inclinometers, a differential capacitive displacement transducer enables detection of even very small deviations [81].

5.1.1. Measurement of positioning errors

For the direct mapping of positioning errors, generally calibrated artefacts or laser interferometers are measured aligned to the axis of interest [81,92,158]. Artefacts used are gauge blocks, step gauges [160], line scales or calibrated scale/encoder systems. Generally, a high density of data points is desirable to detect even high-frequency errors on the scale. An almost infinite sampling rate can be achieved by the use of laser interferometers (Fig. 9) for the measurement of positioning errors.

Yielding a high accuracy on short- and long-machine axes, the stabilized laser interferometer has become most common for measuring positioning errors of machines. Special attention has to be given to the correct mounting of the interferometer: the displacement should be measured between machine table and spindle or probe, respectively [153,149].

5.1.2. Measurement of straightness errors

Mapping the straightness errors of the machine axes requires the measurement of their lateral displacements along the axis travel. Straightness-measurement systems consist of a straightness reference and a displacement indicator [81]. In order to measure the straightness errors with a material reference, the straightedge reference is placed in direction of the machine axis. As a straightness reference, a calibrated ruler or – for long axes – a stretched wire can be used. The axis is then moved while a distance sensor measures the lateral displacement. Possible sensors for this technique include capacitance gauges, electronic gauges or material dial gauges. The straightness standards must be carefully aligned to avoid mixing with other errors. By employing reversal methods (rotating the standard), the calibration errors of the standard can be eliminated. However, gravitational deformation will always point in the same direction [47].

When the linear propagation of a laser beam is employed for determining straightness errors, the displacement between the beam and a detector or reflector is measured. Position sensitive devices (PSD) are commonly used in this context. PSD are electronic detectors that generate an electrical output depending on the position of a laser beam relative to the centre of the sensor. The straightness error can be determined directly, providing the PSD is calibrated properly. Points of concern are the pointing stability of the laser, the PSD resolution and linearity [145] as well as the deflection of the beam due to atmospheric gradients and turbulences [6,31].

Alternatively to the detection by a PSD, a straightness interferometer may be used consisting of a Wollaston prism and a reflector (Fig. 10) [38]. The prism acts as a beam splitter

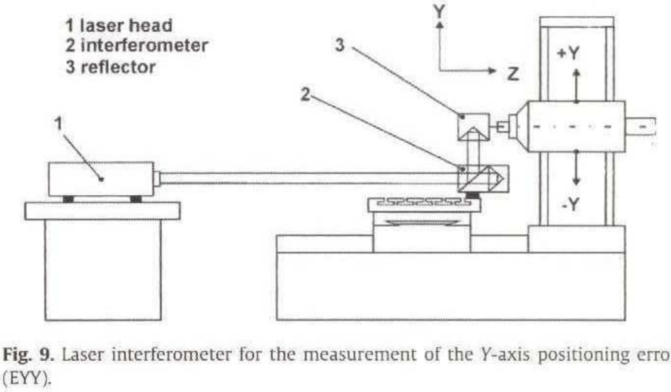


Fig. 9. Laser interferometer for the measurement of the Y-axis positioning error (EYY).

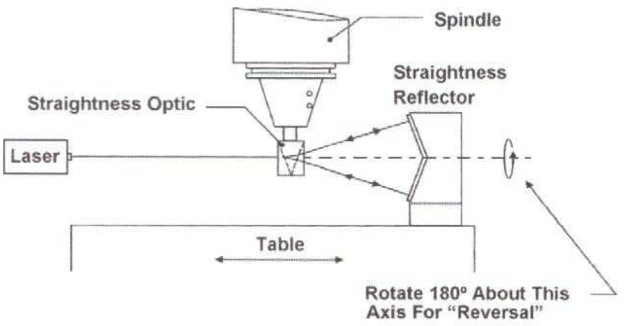


Fig. 10. Straightness interferometer with Wollaston prism. In this setup, the symmetry line of the reflector is the straightness reference.

generating two separate beams that exit the prism at an angle. Both are reflected and recombined to generate an interference signal that allows the lateral displacement of the reflector to be determined. This measures the straightness in one dimension only.

Another method for straightness measurement of components utilizes the direction of the gravitational vector as a reference. Using an electronic level, the angular change of a surface, e.g. a guide way, in relation to the gravity field of the earth is observed. By integrating the angle over the stepwise displacement of the level along the surface, the straightness of the surface can be evaluated. The use of levels generally requires a reference level to be fixed to a non-moving part of the machine to cancel out movements of the entire machine. The differential signal of two levels is very sensitive to non-linearities of both sensors.

5.1.3. Measurement of angular errors

Measurement of angular errors in machine axes is performed either through the use of electronic levels (see above) or laser-based techniques. Standard-based methods are also employed by combining two linearity measurements taken in parallel orientations: one close to and one far away from the axis considered. In such a differential measurement the calibration uncertainty of the standard partly vanishes. Longer measurement time increases the effect of environmental influences [81]. Measurement of angular errors can also be achieved through the use of an angular interferometer (Fig. 11) [30].

Two parallel beams are generated with a beam splitter and are reflected by an assembly of two reflectors mounted in the machine (Fig. 11). An angular deviation results in a path length difference of the two beams. Alternatively, laser interferometers have been designed to operate with three parallel measurement beams, so the positioning error and pitch and yaw errors are measured simultaneously [39]. Another approach is the use of an autocollimator to measure angular errors: A collimated light beam is aligned to a plane mirror that is fixed to the machine axis. The reflected beam travels back to the measurement system where it is detected either visually or with a PSD [114].

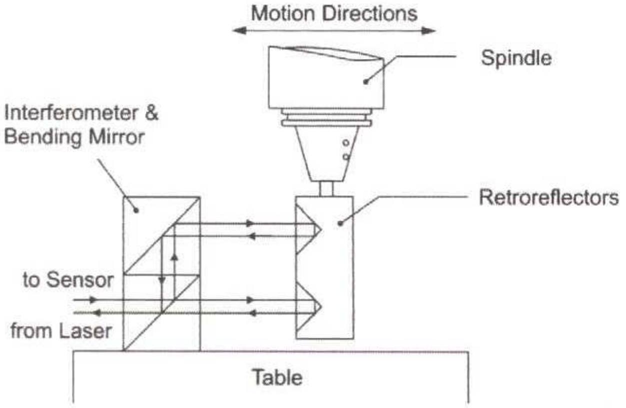


Fig. 11. Angular interferometer.

The rotation around the axis of motion (roll) cannot be measured with an autocollimator or with a laser interferometer with angular optics.

For this, the only known direct method is using electronic levels attached to the axes that measure the rotation directly. Electronic levels can also be used for the other rotary measurements. They have a similar resolution as the optical methods (autocollimator, laser interferometer) but do not depend on any optical path, so they can be used for long strokes or in a bad temperature environment. They usually determine rotations around one axis; however, instruments exist that also measure two rotations at a time [81]. A limitation of electronic levels is the inability to measure rotations around the vertical axis, e.g. ram axis roll on vertical rams.

5.1.4. Measurement of squareness errors

To measure the squareness of two axes, an angle standard, e.g. a granite or ceramic square, can be employed, which has a common frame of reference for the axes [116]. Additionally, laser techniques may be used by performing straightness measurements along two axes. In this case, the laser source remains fixed and one axis is measured, e.g. with a PSD or Wollaston prism setup. Following this step, a pentaprism is placed in the machine into the path of the beam, deflecting the laser by 90° onto a detector placed on the second axis.

5.1.5. Measurement of rotary axes

A conventional method for the calibration of rotary axis is described in ISO 230-1 which proposes the use of a dial gauge to measure the radial and axial run-out deviation at the centre hole of the rotary axis. If the gauge cannot be applied at the centre hole, it can be used in combination with precise manufactured check gauges which are mounted on the rotary axis [81]. Another possibility is the use of capacitive or inductive sensors, which measure contactless and can be used at much higher rotational speed [85,88].

The radial error motions represent two degrees of freedom orthogonal to the rotary axis. To measure this deviation two sensors have to be used for the measurement of the linear displacement, e.g. relative to a cylindrical gauge. This measurement has to be repeated at a second position along the axis in order to evaluate any tilt error motions. The axial error motion represents the axial movement of the rotary axis as its third linear degree of freedom. It is measured by a single sensor which is applied, e.g. in the centre of the front surface of a rotating device. The measurement of the face error motion with the reference plane mounted square to the spindle axis is superposed by the axial error motion and the tilt error motions. Of course, all measurements of the five degrees of freedom can be combined in a single measurement, if multiple sensors are applied at one gauge [40,50,112,143]. The last degree of freedom is the error of the rotation angle itself, which can be measured with a laser interferometer in combination with a self-centring device and the optical components for an angular measurement.

5.2. Indirect measurements (analysis of superimposed errors)

5.2.1. Types of indirect measurements

Indirect measurements require multi-axes motion of the machine under test (e.g. movement to measure positions at different *X*, *Y*, *Z* positions, simultaneous movement of two linear axes, simultaneous movement of linear and rotary axes, machining of a workpiece with several machine axes moved).

Indirect measurements might use artefacts that are partially or totally uncalibrated [11,61], self-calibration methods with or without scale factor (see [116] Sections 2.2.2 and 2.2.3) or calibrated artefacts [158,140,137,103,104,48,19].

On machine tools, special test pieces can be machined, even in the form of artefacts used for a CMM. Then these artefacts are measured on a CMM [119]. However, influences of machining parameters, tool wear, clamping of test pieces on the machine tool and the CMM might increase measuring uncertainties significantly. Therefore artefacts for CMM calibration are also used directly on machine tools and probed with special probing systems.

Another group of indirect measurements, so-called contour measurements, use simultaneous movements of two or more axes to move on special lines, e.g. straight lines in space [95], circular paths produced by two linear axes [110,28,93,32], or circular paths produced by two linear and one rotary axis [90,94,141]. Deviations from the nominal lines are measured with special equipment or relative to an artefact.

Indirect measurements might also use equipment from direct methods (see Section 5.1) and apply the equipment in different positions and orientations in the working volume of the machine, so-called multilateration, e.g. a pure displacement measurement applied in a large number of different orientations in the working volume [37,131,155,123].

A further group of indirect measurements uses simultaneous linear and rotational movements on the machine with nominally no relative movement between the tool/probe side and the workpiece side of the machine, the so-called chase-the-ball measurement. Relative movements, which are caused by any machine error, are measured in one, two or three co-ordinates. One example is testing of a rotary table on a CMM, where the centre of a precision sphere on the rotary table is measured at several angular positions [66]: nominally, the centre of the sphere does not change in the workpiece co-ordinate system of the (rotated) sphere. For further applications on machine tools see [109,156,50].

Several indirect methods have been developed, in a first attempt, as quick checks for an integral machine test giving just a value for path deviation [83] or range(s) of deviation for a tolerance check. In the following, indirect measurements to identify geometric machine errors will be described, generally based on a geometric error model of the machine.

5.2.2. *Identification of geometric errors*

Identification of geometric errors can be achieved by analytical methods or by best fit methods. *Analytical methods* are mainly applied when using calibrated artefacts or self-calibration methods [137,116] or when using contour measurements [83]. The identification of geometric errors is straightforward, e.g. the squareness deviation between two linear axes can be evaluated from positioning measurements at two face diagonals, from a circular test or ball bar test [83], or from a reversal measurement with a ball plate [47]. *Best fit methods* are generally used for geometric error identification if multilateration or "chase-the-ball" measurements are used because a large number of (geometric) errors influence the result. Here, a detailed model of the machine and its geometric errors is needed. Best fit algorithms adjust the model in order to cover the measured deviations. Most models are based on rigid body behaviour assumptions: six component errors per linear and rotary axis, three location errors per linear axis, and five location errors per rotary axis [116]. Defining the machine co-ordinate system including the machine axes leads to the 21 geometric errors of a 3-axis CMM, to 52 geometric errors of a 5-axis machining centre, and 81 for a 6 degree of freedom parallel kinematic hexaglide [128,98]. For this approach (with error model and best fit algorithm) it is essential to distinguish the different geometric parameters. Each error to be identified must have a distinguishable effect on the measurement result and can be separated from any combination of the other parameters. If this is not the case, errors will be identified wrongly, as shown clearly by [129,35,133] for the step diagonal test [148].

On parallel kinematic machines and hybrid machines, direct measurements (Section 5.1) are either not applicable or may be justified for a few geometric errors only. Therefore, generally indirect methods have to be used [14,21,3,36,62,98,43].

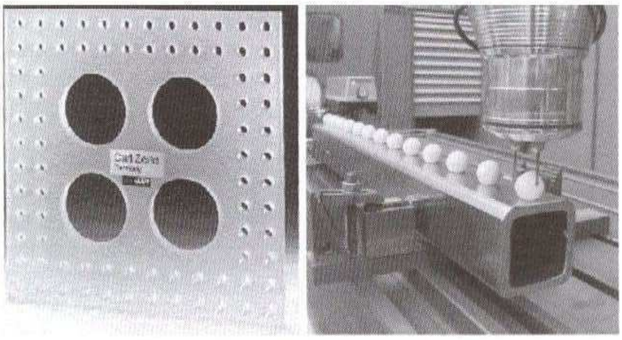

(A) (B)

Fig. 12. (A) 2D artifact with calibrated co-ordinates of holes manufactured from Zerodur [138]. (B) 1D carbon fiber artefact with calibrated sphere co-ordinates [140].

5.2.3. *Example 1: measurement of 1D and 2D artefacts*

An established indirect measuring method is the use of calibrated 1D [158,140] and 2D [137,103] artefacts in different positions in the volume. The artefacts have reference elements that are calibrated in 2 or 3 co-ordinates. By comparing measured co-ordinates to their calibrated values, error vectors resulting from the superimposed kinematic errors of the machine can be detected. By combining the data from several measurements at different orientations, an analytical or a best fit solution can be derived for the single kinematical errors. Although several measurements can also be combined to cover the entire volume, the artefacts size generally has to fit to the workspace of the machine (Fig. 12).

5.2.4. *Example 2: 3D ball plate*

Another example is a kinematic, three-dimensional artefact, the so-called 3D ball plate [19] see Fig. 13. The artefact allows to measure *X*, *Y*, *Z* deviations at several points in the working volume of the machine. These deviations can be used to identify single geometric machine errors with simple formulas, or can be used as direct input for a spatial grid of compensation vectors, without an underlying kinematic model.

5.2.5. *Example 3: indirect measurement based on displacement measurements*

Significant research has been presented on the indirect evaluation of geometrical errors based on displacement measurements [11,127,8,44,48]. For displacement transducers interferometers or ball bars have been used. To achieve a stable solution for all kinematic errors, a huge number of measurements with low-numerical correlations have to be performed. A conventional laser interferometer has a long-working range, but the variation of the

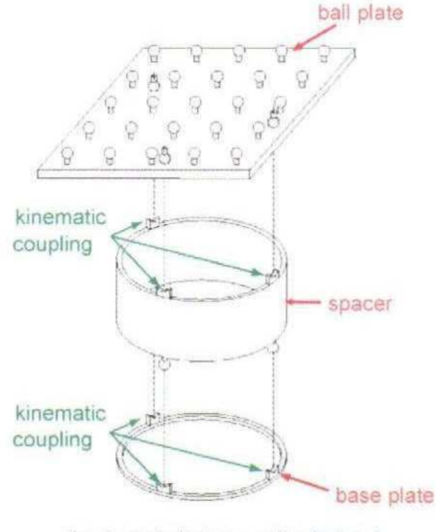


Fig. 13. 3D ball plate artefact [19,125].

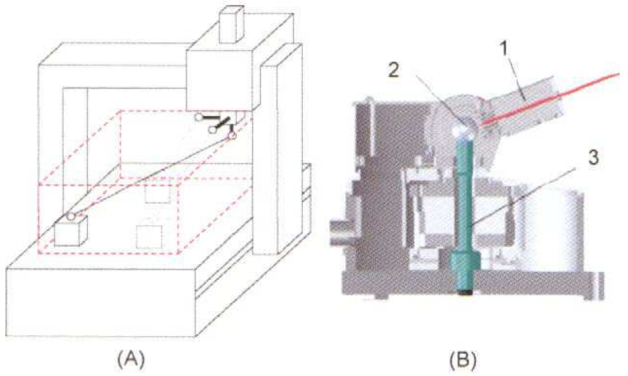


Fig. 14. (A) Multilateration on a 3-axis machine with a Lasertracer [123]. (B) Schematic view of the Lasertracer: (1) tracking interferometer, (2) stationary reference sphere serving as a reference mirror for the interferometer, (3) thermally invariant stem supporting the reference sphere.

measurement direction needs manual interaction [127]. A ball bar can simply generate a number of measurement directions but is limited in the usable stroke [48,131,155]. A combination can be seen in the use of the so-called Lasertracer [123,144] (see Fig. 14). Here a special laser tracker follows the target reflector positioned in a spatial grid and records the spatial displacement. Therefore, a large number of spatial measurements (1000–5000) can easily be generated that covers the entire working volume.

The Lasertracer measures directly in reference to a stationary sphere, which significantly decreases the radial measurement uncertainty of the system compared to conventional Lasertrackers [105,63,123].

A technology based on multilateration with fiber-based white light interferometry has been demonstrated by [124]. The system has demonstrated a good repeatability in small working volumes, and concepts for extending the working volumes have been discussed.

5.2.6. Example 4: *R*-test

The idea of measuring with one or more length measuring devices relative to a precision sphere has been used for a long time. It started with measurements of radial and axial movements of rotational axes [135,24,112,85,142] and with single point run out measurements for the location of rotary axes [40,5]. Another group of measurements with a similar setup are (thermal) drift tests, where drift in X, Y, Z is measured against a precision sphere [4]. Testing of rotary axes on a CMM [66] uses a moving precision sphere measured in several positions of the rotary table. This idea was transferred to 5-axis machine tools, where combined movements of rotary and linear axes keep the distance between tool and workpiece side nominally constant. Relative movements between tool and workpiece side are measured with a precision sphere and three or more length measuring devices [109,156,50,22]. One possible set up is shown in Fig. 15.

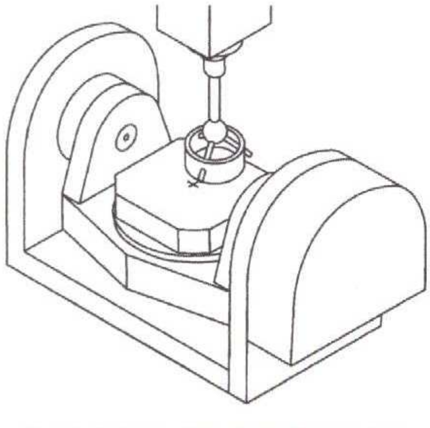


Fig. 15. *R*-test on tilting rotary table machine.

Three different setups with kinematic ball bars, one setup with the ball bar measuring radial to the axis of rotation, one with the ball bar in a tangential direction, and one with the ball bar in the axial direction, in principle give the same result as an *R*-test [5,141].

5.3. Uncertainty of measured error parameters

5.3.1. General considerations

When reporting the error parameters of a machine, an uncertainty must be connected to the reported numbers. Thus the usefulness of the measurements can be determined, parameters can be compared to their specifications, taking the measurement uncertainty into account. Any uncertainty calculation should follow general guidelines, e.g. the GUM [55] or the PUMA method [79]. Due to numerous existing measurement techniques (see previous sections), uncertainty calculations cannot be given in detail here. Nevertheless, some general critical aspects of the uncertainty calculations of the measurements described in the preceding sections will be discussed.

5.3.2. Machine repeatability

The repeatability of the machine, usually expressed as a standard deviation, is a part of any uncertainty budget. For estimation, several repeated measurements of a specified parameter are needed. Most standards and guidelines prescribe five repeated measurements or more. As the machine will later measure (or make) a workpiece just once, the standard deviation in a single measurement should be derived from the measurement dispersion, and not the 'standard deviation of the mean'.

5.3.3. Uncertainty in the used standard

Generally, all used standards must be calibrated and a calibration report must indicate an uncertainty. Not only gauge blocks, but also laser interferometer systems, straightness optics, autocollimators and straightedges need calibration and the calibration already may have an uncertainty that is close to the tolerance of the measured parameter. If a standard is used in such a way that its calibration value cancels out by reversal methods [47] then its uncertainty may vanish as well.

5.3.4. Uncorrected deviation of a standard

For a gauge block or a step gauge it is trivial that the real calibrated value must be taken into consideration, and not its nominal value. For other standards, this is less trivial and can be less easy to perform; for example, a systematic deviation of a laser interferometer system due to a deviation of a temperature sensor, while the laser interferometer is used in an automated machine evaluation system that does not allow for corrections. Another example is the straightness deviation of a straightedge while the machine cannot correct this point by point.

5.3.5. Uncertainty due to misalignment

A proper alignment must minimize Abbe and cosine errors. Sometimes alignment is more critical than one might expect. For example, a straightedge that is slightly misaligned along an axis might pick up interpolation errors of the linear scale that measures the deviations; this might appear as long-wave straightness errors.

5.3.6. Uncertainties in direct measurements

Direct measurements can be directly related to a single error source in terms of the kinematic model explained in Section 5.1. Scale deviations should be measured in line with a scale (Abbe principle), but it is generally agreed that also a linear positioning measurement close to a scale can be considered as a direct measurement, provided that measurements of rotary errors show that these have a negligible effect on the positioning error measurement (EXX, EYY, or EZZ). The 'directness' of a direct measurement can only be approximated by reducing the other error sources. Also "direct" measurements will generally measure superimposed errors, like the example given in Section 5.1: for measuring the straightness of an axis by measuring a straightedge, also the scale of the perpendicular axis is involved. When probing, especially from different sides, the probing error is also involved.

5.3.7. *Errors in calibration of linear positioning*

Apart from the effects mentioned above, environmental effects, especially temperature, may effect these measurements. If the machine scale is calibrated using a material standard (gauge block, step gauge), temperatures different from 20 °C, temperature differences, and differences in expansion coefficients between the used reference and the scale to be calibrated cause linear deviations in the measured points.

The effect of a difference in expansion coefficient between scale and reference standard is proportional to the (average) deviation from 20 °C, and the effect of a temperature difference is proportional to the average expansion coefficient. The uncertainty in the temperature does not only depend on the calibration of the temperature sensors; it is also a matter of the temperature sensors acquiring the temperature of the used gauge and the scale properly. For limiting this uncertainty contribution, it is an advantage when low-expansion materials are used for both the scales and the reference artefacts: the manufacturer can both claim and prove low uncertainties for the positioning errors. However a user measuring a steel artefact on such a machine loses much of the gained uncertainty in the measurement and estimation of the artefact's temperature and expansion coefficient, especially when not measuring at 20 °C. More details and equations on these aspects are given in [80].

In case a laser interferometer system is being used as reference, the sensitivities of the air refractive index for environmental conditions are summarized in Table 1.

Also, a low-expansion coefficient reduces the influence of material temperature uncertainty. However, the influence of the air temperature is always present and results will depend on a stable environment rather than on the uncertainty of the air temperature measurement system. Air temperature sensors that produce heat shall be placed away from scales and laser beams, e.g. above. Pitch and yaw deviations (originating from other error sources than the one being measured) may affect the measurements if these are rather large. Detailed equations and examples are given in [165]; most of these also apply to CMMs. A characteristic of linear scales is that they may have interpolation errors that repeat periodically (e.g. at a pitch of 80 μm). This may give apparent long-wave errors if the positioning error is measured at equidistant locations (of the Nyquist criterion), or may hide the problem if the calibration locations coincide with exact multiple pitch distances.

5.3.8. *Straightness calibration*

The uncertainty of a straightness calibration typically depends on the repeatability of the measurement and the calibration of the straightness reference. In this case, temperature gradients can cause a bending of the machine axis and the standard. If a laser interferometer is used with special optics for straightness, its optics must also be calibrated. The absolute flatness of the reference mirror is of high importance. As this flatness deviation enters into the measured straightness with a factor of $1/a$ (typically a factor between 30 and 50), even a low uncertainty of $\lambda/10$ already results in an uncertainty of about 1 μm in the measured straightness. Because of the same amplification, the sensitivity to air turbulence is much higher than in a displacement measurement. Another significant uncertainty contributor can be the refraction of the laser beam due to a temperature gradient perpendicular to the beam [4].

Table 1
Sensitivity of laser distance measurements to environmental parameters.

Condition	Uncertainty	Resulting uncertainty
Air temperature	1 °C	1 μm/m
Air pressure	1 hPa	0.3 μm/m
Humidity	10% RH	0.1 μm/m
CO_2 content	100 ppm	14 nm/m

5.3.9. *Calibration of rotations*

As usually only small angular deviations are measured optically, the calibration of the angular optics, i.e. the determination of the distance between the prisms in Section 5.1, is not so critical. Usually reproducibility is a major factor in the uncertainty. For larger distances, the turbulence in the laser beams may limit this reproducibility. When using (electronic) levels, the linearity between reference and measuring level becomes crucial. Over large distances (several meters), the curvature of the earth's gravity field must be taken into account [150].

5.3.10. *Squareness calibration*

The squareness of the used standard and its uncertainty are usually dominant factors. When using optical squares, care must be taken to correct their deviation with the correct sign, and one must be aware that even a rather accurate calibration, e.g. with an uncertainty of 0.5″, still corresponds to an uncertainty of 2.5 μm/m.

5.3.11. *Uncertainty in indirect measurements*

If an axis deviation is evaluated from a combination of measurements, the uncertainty in this deviation and its later use in determining a positioning uncertainty may become a complicated issue due to correlations in the measurements. Examples of these effects are given by Florussen et al. [48] for deriving geometrical machine errors from ball-bar measurements. An example is the derivation of both the rotation and the positioning error along one axis by positioning measurements at different distances from one axis. Generally, great care has to be taken to avoid correlations between single errors by a large number of linearly independent measurements. One way of recognizing these interdependencies was demonstrated by [123,22] by using Monte-Carlo simulations. Simulations can be regarded as a powerful tool for optimizing indirect measurements in regard of the resulting uncertainties.

5.3.12. *Correlation between straightness and rotations*

In the machine model, the straightness and rotations may be independent, but because of the mechanical construction of the axes, they will be correlated: if an axis is not straight, it will cause rotations, although mostly not in a straightforward manner. This may lead to an over-estimated uncertainty if the uncertainties of both straightness and rotations are combined.

There is another obvious correlation between translations, rotations and squareness: the squareness as a separate parameter in the model correlates with a constant term in the rotations and a linear term in the straightness errors (although straightness is defined as not having a linear term, see ISO 230-1 [81]). This means on the other hand, that only one of these parameterisations should be chosen: if the squareness deviations are taken as separate parameters, then the constant term in the rotation must be zero, and the straightness should not have a linear term (as already by definition) [7].

5.3.13. *Other correlations in parameter estimations*

If a complicated machine model is fitted to a limited amount of measurements, correlations can have a major influence if the parameters must be derived from complicated combinations of measurements. This is especially the case when only length measurements are taken, such as a Lasertracer, or from, e.g. ball bar measurements. The mathematical fit to a machine model can be done effectively by standard least-squares techniques, however the user must be critical on the result of these calculations; e.g. it should be considered whether the resulting machine parameters are realistic.

As an example: ball bar measurements are not so sensitive for straightness deviations. This means on the other hand, that the least-squares evaluation easily attributes major values to the straightness deviations to compensate for small measurement deviations. If the calculation gives unrealistic values for parameters, the best measure is to leave these parameters out of the model and accept a less perfect fit. Unrealistic parameters would not represent a big problem for similar measurements, but give major deviations for different trajectories in the measurement volume, e.g. a straight line along an axis. The correlations also mean that a bad repeatability in one axis can appear as an apparent bad repeatability of other axes.

Another example is a simple circular test that is used to identify squareness between X and Y by comparing the diameter of a circular standard measured at +45° and −45° [83]. The result gives the squareness deviation on a perfect machine (and with a perfect measuring instrument). However, local positioning errors, backlash, local straightness deviations, local pitch, yaw and roll movements of the moving axes X and Y influence the evaluated diameters and the resulting squareness deviation, so they correlate with the squareness for this (limited) measurement. In this case the correlation is also due to an incomplete model.

5.3.14. Incomplete model

An incomplete kinematic model may cause correlations or just erroneous estimations. The incompleteness is not necessary skipping some of the translational and rotational terms, but it can also be the simplified assumptions of the error behaviour: e.g. scale errors that are described by a few low-order polynomials while in fact the major error is a short-wavelength periodic deviation. Other error sources that do not fit in a standard geometric model are backlash and hysteresis. They usually affect uncertainties in the measured parameters since they are not modelled adequately.

5.3.15. Monte Carlo estimations

As it becomes clear from the arguments given in this section, setting up an uncertainty budget in the classical sense is almost impossible in the case of indirect measurements, at least when these are used as more than just a performance indicator. A Monte Carlo uncertainty estimation is often the most appropriate way to get any uncertainty estimation. Such estimation may also help in finding an optimal measurement strategy for measuring parameters with a reasonable uncertainty in a reasonable time.

In this method, a geometric error model is needed, as well as knowledge (or assumptions) on possible geometric errors (limits on ranges and error functions).

For the simulation, one set of geometric errors within the given limits is selected. Then the measurements used for the identification of geometric errors are simulated in the computer, and with the simulated measurements the identification of the geometric errors is carried out. Then the simulated geometric errors are changed (within the known or assumed limits) and the simulated measurements and identification are repeated. After, e.g. 1000 repetitions, a distribution of the evaluated geometric error parameters is generated, which is expressed as an uncertainty parameter such as a confidence interval or a standard deviation. This method has been demonstrated by Schwenke et al. [123] and Bringmann [22].

6. Performance criteria of mapping methods

Because of the variety of available technologies for error mapping and the varying requirements arising from different applications, the best technology for a certain application has to be carefully chosen. The following sections review the main criteria.

6.1. Universality/scalability

The first step is to define the range of machines that has to be covered: the axes travels, the number and the type of axes (translation/rotation) as well as the kinematic structure. For example, some methods based on artefacts and telescoping ball bars are not suitable for long axes, while most indirect methods require the kinematic structure available in the respective software library. Also the achievable sampling density must be adequate for the wavelength of errors that is expected in the guideways.

6.2. Completeness of information

In a next step, the error parameters relevant for the machining or measuring process must be identified. This requires an analysis of the machine structure and of the intended machining or measuring process [107]. For machine tools with an approximately constant tool centre point or for an optical CMM without a changing probe offset, the rotational errors of the last axis are not relevant for the compensation. In contrast, these might be very critical parameters for other machines (e.g. for CMMs used with long-probe offsets). For 5-axis machines, especially the location errors (position and orientation) of the rotary axes are critical, while for large horizontal arm machines (CMM and machine tools) some elastic errors might be dominant.

6.3. Uncertainty

Since a volumetric error mapping requires the measurement of different error types (positioning, straightness, pitch, roll, yaw, radial, axial and tilt error motion), the uncertainties have to be looked at separately for each (see Section 5.3). The measurement uncertainties for direct measurement setups are generally specified by the manufacturer of the metrology equipment. These specifications may include a length dependent term. Mostly, the achievable uncertainties are very dependent on the environmental conditions and the manufacturer's specifications have to be carefully reviewed under non-laboratory conditions [97]. This is especially true for systems that use a laser beam as a straightness reference. The uncertainty for systems based on indirect measurement depends strongly on the machine and the setup and cannot be specified generally (see Section 5.3). The calculation of the uncertainty will demand specific modelling and evaluation, e.g. based on automated Monte Carlo simulations [122,123,22,167].

6.4. Operator influence

Most technologies available require an operator with significant knowledge and experience. The quality of compensation will largely depend on his understanding of possible uncertainty contributors, e.g. alignment or environmental issues. These experienced operators often present a high-value resource for the machine manufacturer or service provider. But increasingly, systems and their respective software envisage tools to facilitate alignment and data interpretation. Some systems have minimized the effort for alignment and manual operations and offer automated procedures and data evaluation with a minimum of operator interaction. Although data acquisition can be performed by regular machine operators, a fundamental knowledge on error compensation is necessary to deal with problems and unexpected results.

6.5. Measurement time

As [116] states, the time required for the measurement of all systematic errors is one major barrier to employ error compensation techniques for machine tools. All mapping procedures include three phases:

(A) *Setup phase.* This is a fully manual operation. Depending on the method, it may require fine alignment of the instruments and a skilled operator.

(B) *Data acquisition phase.* The machine is typically operated in CNC mode. Depending on the method and the interfaces, this phase may require a different amount of operator interaction.

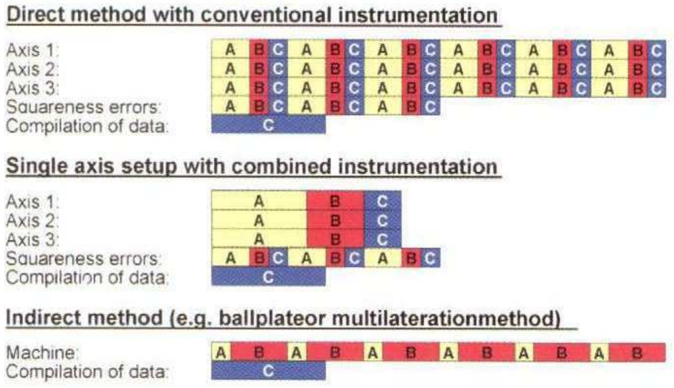


Fig. 16. Comparison of mapping methods for a 3-axis machine: (A) setup phase, (B) data acquisition phase, and (C) evaluation phase.

(C) *Evaluation phase.* The results are interpreted and combined to yield a complete error map.

Fig. 16 shows typical general sequences of these phases for direct and indirect methods. Development of combined instrumentation (see Section 5.1) has reduced the overall time for direct measurements significantly. For indirect methods like the ball plate and the multilateration method, the measurement time is dominated by the data acquisition, which can be unmanned [48]. Furthermore, in the case of the multilateration method the setup time is reduced significantly: accurate alignment of the instrumentation is no longer necessary.

6.6. Interfaces

Each error mapping method needs to interface with the machine controller during the data acquisition and, at the end of the procedure, has to generate the compensation data (see Fig. 17). Different techniques are used for the data acquisition:

(1) Generation of machine code and automatic standstill detection by the metrology software.
(2) Direct interface, e.g. by a trigger signal or a bus protocol.
(3) Use of analogue or touch trigger probes when probing an artefact.
(4) Direct readout of the analogue scale signals by the metrology PC.

A critical point is the generation of the look-up tables or compensation functions. While the format of the compensation data for machine tool is mostly documented and accessible, many compensation data files for CMM's are protected by the CMM manufacturer. The availability of tested interfaces and data formats is crucial for a successful implementation of mapping methods.

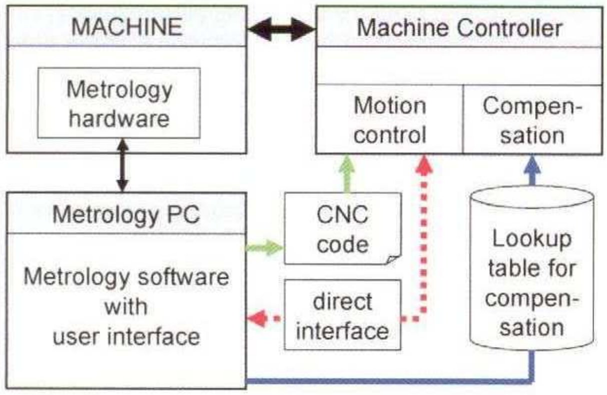


Fig. 17. Interfaces between the metrology for error mapping and the machine. Green: generation of CNC code for data acquisition. Red: direct interface for machine control for data acquisition (alternative). Blue: generation of look-up table for compensation.

6.7. Traceability

The traceability of an error mapping process demands the traceability to the meter of all measurement processes involved. Strictly speaking, all metrological components involved in the measurement must undergo calibration, including the sensors and the optical components (see Section 5.3). For setups that only measure small deviation from a nominal zero, the requirement for the scale error is relatively reduced. But for rotational measurements with differential electronic levels, even small linearity errors between the two systems may result in rather large relative errors. Indirect measurements, which use a single artefact (e.g. a ball plate) or only displacement measurements (e.g. articulated ball bar, Lasertracer) may have an advantage in regard to traceability, since they only demand a single artefact or instrument calibration.

6.8. Additional considerations

Systems, which directly measure the physical rotation of a carriage (e.g. by angular interferometers, see Section 5.1) will not detect the difference between a compensated and an uncompensated machine, when the compensation is limited to linear effects due to roll, pitch and yaw error motions. Therefore, they are not suited to check machines already compensated, while indirect methods will generally be sensitive to compensations already applied. This aspect is also mentioned by [116]. On the other hand, direct methods might be superior, when mechanical alignments have to be made. Once aligned, they can give direct feedback to mechanical adjustments for a single axis.

Great care has to be taken for the synthesis of directly measured data to a complete error map. The boundary conditions for each error have to be defined precisely to avoid crosstalk between error functions. For indirect measurements, the solution usually is the result of a global fit of all parameters; therefore the result is a consistent set of parameters.

7. Numerical compensation

The previous sections introduced kinematic chains and summarized the available technologies for the determination of geometry errors. This data can either be used for mechanical fine alignment of the components or for creating numerical data to compensate the systematic errors during the measurement (CMM) [106] or machining (machine tool). Hocken defined the term software correction as follows: "Software correction is the use of pre-process data, a machine model, and indirect sensing of process parameters relevant to that model, in order to provide data to the control system for the correction of a nominal tool position with respect to a nominal part during the process of machining or measuring using the actuators 'normally' supplied with the machine." [59] Instead of using the term "software correction", in the following the term "numerical compensation" is used for consistency.

After first scepticism of many manufacturers and customers, today the compensation has been accepted by the market as an integral component of a modern CMM. The consequent use of compensations also had an impact on the production of CMMs: the design and manufacturing of measuring machines today is mainly focussed on high repeatability, including thermal stability and minimized hysteresis. The requirements for absolute mechanical precision of the guideways can be greatly relaxed due to the mathematical compensation that is generated and applied after final assembly of the CMMs. Today, high-end CMMs with maximum permissible errors (MPE) below 3 μm are on the market which, if uncompensated, would have MPEs of 100 μm or more [139]. Only very few CMMs are available today without an extensive geometric error compensation. To produce highly accurate horizontal arm machines, the model was extended to cover some so-called elastic errors of these CMM [9].

Sartori and Zhang recognized three ways for the numerical compensation of errors:

1. Continuous compensation in the path generation of the CNC controller (real-time transformation of interpolated points).
2. *End point compensation.* Geometry errors still appear in the path.
3. *Final result compensation for measuring machines.* Only the result of measurement is corrected by the known deviation.

For machining of parts, the first option is clearly favourable to generate accurate features and surfaces. Applications have been demonstrated, where an end point compensation was realized by a NC-code transformation [130]. To reduce the problem of remaining errors in the path, nominal movements can be incremented by single commands, where the end point compensation is applied to each single incremental movement.

In the case of a compensated path generation, the effect of translational and rotational errors on the position of a tool reference point can be fully compensated [15]. In case of a milling machine with a spherical milling head, where the physical orientation of the tool does not affect the machining result, full compensation of all errors on a 3-axis machine is possible.

In the more common case, that a tool is not spherical but cylindrical or flat, the physical orientation of the tool will still affect the machining result also in a compensated 3-axis machine. Fig. 18 visualizes the effect on the milling of a plane.

Even in the case, in which the path of the tool centre point on a 3-axis machine is perfectly compensated for the ram axis errors, some errors in the shape of the plane remain. Only for a 5-axis machine the physical rotations of the tool can be fully compensated, too. But it has to be considered that, due to the distance between the tool centre point and the rotary axes location, even the compensation of small angles may require a significant additional motion of the linear axes. Therefore, residual errors of all linear and rotary axes may generate additional errors not present for a standard 3-axis machining operation. A typical cycle time of a modern CNC controller is 2 ms or less. During this time the interpolated trajectory points have to be transformed according to the kinematic model and the respective compensation parameters. Alternatively, a pre-processing of the tool path can be performed in so-called *soft real-time*.

Some compensation methods are not based on single geometric errors of the machine axes, but directly compensate *X*, *Y*, *Z* deviations in the working volume of the machine, applying an error lattice [116]. In this case no model is needed. This approach requires data acquisition in the entire working volume, which necessarily results in long-measurement times. However, in this case, only translational deviations, which may be caused by linear or rotational error motion, can be compensated.

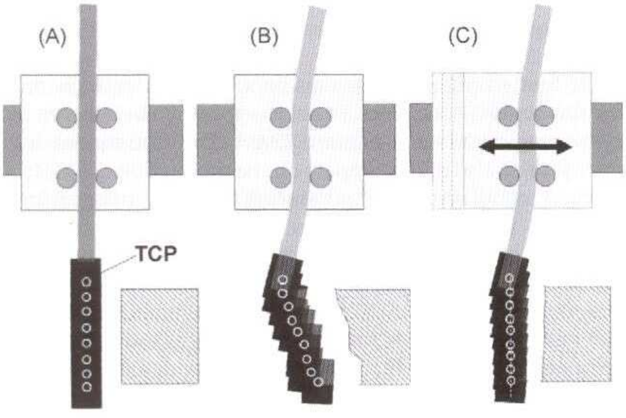


Fig. 18. Effect of compensation demonstrated on the slab milling of a plane: (A) machine with no geometry errors of ram, (B) uncompensated ram geometry errors, and (C) compensated path of the tool centre point (TCP), but physical rotation of the tool due to pitch and yaw not compensated.

Recent work on volumetric compensation for machine tools by Longstaff, Fletcher and Myers [168,169], for both geometric and thermally induced errors of 3-axis and 5-axis machines has resulted in geometric errors been reduced by as much as 97% and thermal errors by 75%. The compensation has been applied by integration directly into CNC controllers and also by external means through separately mounted computers situated in feedback circuits and has been integrated successfully into both small and large machining centres.

Nevertheless, applying numerical compensation to machines with noticeable hysteresis or insufficiently trimmed controllers may also have negative effects. In a compensated machine, all axes are generally moving, even if the nominal tool path would not require such movement. For example, for machining an *XY*-plane, the *Z* axis will move continuously in small increments and changing directions to compensate for straightness deviation of the *X* and *Y* axis. This motion in combination with a hysteresis of the axis may harm the surface finish quality. With modern controllers and current machine designs, this effect for most applications will generally not be recognizable.

8. Performance evaluation in standards

8.1. Acceptance testing and re-verification

8.1.1. Standards for CMM

Effects of geometric errors and of single point probing are evaluated, when measuring block gauges or step gauges in several directions in the working volume of the machine [34,65]. In the upcoming revision of this standard, the use of laser interferometers and ball bars will also be included. Checking the (geometric) performance of a rotary table is defined in [64] and already discussed in Section 5.2. The problem of estimating the uncertainty of the above-mentioned tests is covered in [86]. The testing of single parametric errors of CMM's is not described in an international standard; the only available public document covering this subject for CMM's is the German guideline VDI 2617-3 [146].

8.1.2. Standards for machine tools

The basic standard for geometric testing of machine tools is ISO series 230 [81–85] which is under revision in order to include new measuring instruments and new kinematic tests, like the *R*-test. The basic standard for positioning linear and rotary NC axes is [82], for geometric tests of axes of rotation it is [85], for circular tests [83], for diagonal displacement tests [84]. These basic standards can be applied to any machine tool. However, for several machine types there are machine specific standards, which also include tolerances for the measured deviations. An important series of machine specific standards is ISO series 10791 for machining centres, with geometric tests for horizontal machining centres [67], vertical machining centres [68] and for vertical machining centres with universal heads [69], with positioning tests [70], tests for interpolation [71] and for contouring performance [72]. The part for testing interpolation is under revision [164] in order to include kinematic tests for 5-axis machine tools.

Another important series is ISO 13041 for NC turning machines and turning centres, with geometric tests for horizontal turning machines [73], vertical turning machines [74] and for inverted vertical turning machines [75], with positioning tests [76], with tests for interpolation [78] and for contouring performance [78].

Testing of 5-axis machine tools today is not very well covered in ISO, but the working group on machining centres started to revise existing standards and to produce a new standard especially dedicated to geometric testing of 5-axis machining centres.

8.1.3. General standards

Standards of general interest, when geometric testing machines, are the influence of temperature on length measurements ISO/TR 16015 [80] a technical report, and procedures for thermal testing of machine tools ISO 230-3 [161].

8.2. *Interim checks*

[65] includes interim checks for CMM's, like measurement of precision spheres or customized reference artefacts. The kinematic tests for machining centres and turning centres [71,77] are typical interim checks, as well as the tests for the contouring performance [72,78]. Another group of checks, which can be easily applied for interim checks, is machining of test pieces, e.g. on machining centres [162] and on turning centres [163]. In the workshop, often interim checks are used to check the location of rotational axes on machine tools, especially before a critical part has to be machined. This can be done by probing special surfaces on rotary tables with the touch trigger probe generally used to measure the location of a (pre-machined) workpiece on the machine tool. Another approach is machining two small planes at 0° and 180° position of the rotary table and taking the distance for checking the position of the axis of rotation. A further approach uses the *R*-test for automatic recalibration of the axis of rotation, provided the NC allows inputting all evaluated parameters.

9. Summary and conclusion

While full error compensation for CMM's has been an established practice for many years, its application is increasingly extended to machine tools. Higher accuracy demands and also the intention to decrease the costs of mechanical accuracy promotes this development. More instruments, tools and software solutions are available to support machine builders and users to map and – in a second step – compensate systematic geometry errors. Consequently, knowledge on machine tool metrology and the application of compensations becomes essential.

The paper has reviewed the fundamentals of kinematic modelling. Various technologies to measure the errors of machines are reviewed and their basic characteristics are discussed. Direct and indirect methods and their characteristics in the practical application have been compared.

Special emphasis has been placed on the uncertainties involved in the measurement of geometrical errors. Typical uncertainty contributors related to artefact based, optical and gravity-based methods have been discussed.

While full numerical error compensation for CMM's is well established, the possibilities for machine tool controllers are often still limited. Today, only a few controllers allow for the compensation of all rotational errors of axes.

However, it can be assumed that the use of numerical compensations will increase in the future. This is promoted by the market requirement of higher accuracy to cost ratio and the development of user friendly and accurate machine tool metrology. Also the current international standards on acceptance testing of machine tools are increasing the customer awareness for this subject, e.g. by diagonal testing of machine tools according to ISO 230-6 [84]. Furthermore, the computing power of modern controllers makes the real-time compensation possible without spoiling the dynamic performance. For these reasons, full error compensation for machine tools is on its way to become a standard procedure for the high-end machine tool market. Today's pioneer applications for this technology are large machines in the aerospace and defence industry but also small high-precision machines, e.g. for die and mould industry or for precision machine manufacturing (e.g. automotive engines). The maximum benefit of a numerical compensation can be achieved on a 5-axis machine, when the rotational errors between the tool and the workpiece can also be fully compensated.

Nevertheless, compensation of machine tools must always deal with geometrical errors changing as a result of thermal changes and load effects. Thermal and mechanical stiffness therefore remains a primary design criterion for machines. Furthermore, the paper mentioned possible negative effects of numerical compensation, e.g. in case the machine has significant hysteresis effects.

However, the achievable benefit will nevertheless generate a significant market for error compensation. The authors estimate that in 2012, 30–50% of all new machine tools will be compensated for positioning, straightness and rotation. Since machine geometries are changing over time in the field, recalibration of machines will not only be offered by machine manufacturers, but also by qualified service providers with dedicated equipment. Consequently, error compensation will also increasingly become a service business. For the development of market and technology, the availability of proper interfaces for metrology equipment as well as the access and the documentation of the respective compensation data will be of crucial importance.

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